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Recent advances in the three flavor Larkin-Ovchinnikov-Fulde-Ferrell phase of QCD

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We present a summary of the recent advances achieved in the study of three flavor Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) phase of QCD. We have explored, using a Ginzburg-Landau expansion of the free energy, the LOFF phase with three flavors, in the simplest single plane wave structure, using the NJL four-fermion coupling. We have found that this phase does not suffer the chromo-magnetic instability problem. A preliminary study of astrophysical effects of quark matter in the aforementioned phase has been done and we have evaluated self-consistently the strange quark mass extending the pairing ansatz to the CubeX and 2Cube45z structure. Finally we have investigated the possibility of Goldstone bosons condensation in the favored cubic structures of LOFF phase.

I. INTRODUCTION

The comprehension of the structure of the QCD phase diagram is one of the most challenging topics within the elementary particle physics. In particular, the study of the region corresponding to very low temperatures and high densities (roughly from 3 to 10 times the nuclear saturation density) had a big impulse some years ago when a Cooper pairing among quarks, driven by the strong interaction, was hypothesized to yield a collective phenomenon named Color Superconductivity (CSC) [1]. At asymptotical densities, the ground state of quark matter is successfully described by the energetically favored phase named Color-Flavor-Locking (CFL) [2] in which all the light quarks u, d, s of any color form Cooper pairs with zero total momentum and all fermionic excitations are gapped. At intermediate densities, where it is not possible to neglect the strange quark mass and the mismatch $\delta\mu$ in the quark chemical potentials due to β-equilibrium and color and electrical neutrality, the situation is much more involved. The ground state of matter in these conditions is still matter of debate and several possible states have been suggested. In particular the superconductive phases characterized by gapless fermionic excitations, gapless-2SC (g2SC) [3] and gapless-CFL (gCFL) [4] have been widely discussed. However, it has been shown that these gapless phases suffer chromomagnetic instability [5] due to the imaginary Meissner masses of some of the gluons associated with broken gauge symmetries (an instability is present also in 2SC phase [6]).

Another possible phase largely discussed in literature is the Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) phase [7]. This phase is relevant since for appropriate values of $\delta\mu$, it can be energetically favored to form Cooper pairs with non-vanishing total momentum $\mathbf{p_1} + \mathbf{p_2} = 2\mathbf{q} \neq 0$, see [8] and for a review [9]. The two flavor LOFF phase has been found energetically favored with respect to the 2SC phase [10] and characterized by real gluon Meissner masses [11]. Anyway a more realistic description of QCD at intermediates densities requires that all the three quarks u, d and s should be taken into account, leading to a theoretical study of the three flavor LOFF phase of QCD.

In connection to this necessity we present, in the following, some important phenomenological results of the recent theoretical studies concerning the three flavor LOFF phase.

II. THREE FLAVOR LOFF PHASE WITH SINGLE PLANE WAVE STRUCTURE

The first work, whose results we would like to show, about the three flavor LOFF phase [12], considered the following ansatz for the spatial dependence of the order parameter:

$$< \psi_{i\alpha}\, C\, \gamma_5\, \psi_{\beta j} >= \sum_{I=1}^{3} \Delta_I(\mathbf{r})\, \epsilon^{\alpha\beta I}\, \epsilon_{ijI} \qquad (1)$$

with

$$\Delta_I(\mathbf{r}) = \Delta_I \exp\left(2i\, \mathbf{q_I} \cdot \mathbf{r}\right) \ . \qquad (2)$$

The three independent functions $\Delta_1(\mathbf{r})$, $\Delta_2(\mathbf{r})$, $\Delta_3(\mathbf{r})$ describe respectively $d - s$, $u - s$ and $u - d$ pairing. The same convention holds for the wave vectors $\mathbf{q_I}$. This means that all the pairs choose the same direction for their wave vectors, so being all parallel or antiparallel relative to one another.

The results of [12] show that there is a window 128 MeV $< m_s^2/\mu < 150$ MeV where the LOFF state has a lower free energy with respect to the normal phase and homogeneous color-superconductive phases. The energetically favored structure has $\Delta_2 = \Delta_3$ and $\Delta_1 = 0$,

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and $\mathbf{q_2}$, $\mathbf{q_3}$ parallel. The study is performed within a Ginzburg-Landau expansion of the free energy up to the Δ^4 order. The interaction between quarks is considered as a NJL four-fermion coupling in the mean field approximation. The strange quark mass is treated as a shift of the corresponding chemical potential: $\mu_s \to \mu_s - m_s^2/(2\mu)$. Finally the β-equilibrium and color and electrical neutrality have been imposed. The validity of the GL analysis with the above approximations has been confirmed in [13].

A. Chromomagnetic stability of the LOFF phase

The introduction of a non-zero strange quark mass, together with the aforementioned neutrality conditions, leads to a thermodynamic system of quarks with different chemical potentials, and this in turns gives gapless dispersion laws. This was first observed in other (homogeneous) color superconducting phases, like gCFL and g2SC, and in both cases it was proved to drive the so-called chromo-magnetic instability of the system, meaning that the screening Meissner masses of some gluons are imaginary [14].

The problem which comes next in the analysis of three flavor LOFF is then to find out whether such a phase is chromomagnetically stable. This analysis has been performed for the three flavor LOFF phase with single plane wave structure [15], and the results are shown in Fig.1. The plotted quantities are the squared screening Meissner masses of the gluons as functions of m_s^2/μ. They are tensors with longitudinal and transverse components and a nontrivial color structure, defined as the opposite of the spatial component of the polarization tensor in the static limit

$$\left(\mathcal{M}^2\right)^{ij}_{ab} \equiv -\Pi^{ij}_{ab}(p_0 = 0, \mathbf{p} = 0) \ , \qquad (3)$$

where (i, j) are spatial indices and (a,b) are adjoint color indices. The gluons Meissner masses are evaluated in Ginzburg-Landau approximation to the order Δ^4.

All the masses are positive, hence the three flavor LOFF phase is chromo-magnetically stable in the Ginzburg-Landau limit. Furthermore the 3-flavor LOFF phase develops the Meissner screening effect along the q-direction, being the transverse Meissner masses suppressed as $\Delta^2/\delta\mu^2$ in comparison with the longitudinal ones. It's worth to note here that although we have computed the Meissner masses only for the single plane wave Fulde-Ferrell (FF) structure, we know from the two flavor case that more complicated crystalline structures have a lower free energy than the FF state and the same is true in the three flavor case, as we will show in the next section. In the general case one should replace (2) with

$$\Delta(r) = \sum_{m=1}^{N} \sum_{I=1}^{3} \Delta_I \exp\left\{\vec{q}_I^{\ m} \cdot \vec{r}\right\} \epsilon_{ijI} \epsilon^{\alpha\beta I} \qquad (4)$$

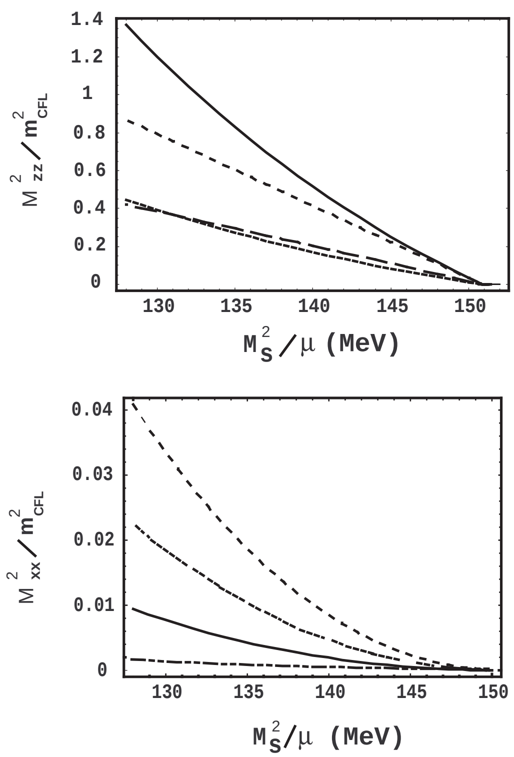

FIG. 1: On the top: Longitudinal squared Meissner masses, in units of the CFL Meissner mass, vs M_s^2/μ; from top to bottom the lines refer to the gluons A_3, $A_6 = A_7$, A_8 (long dashed), and $A_1 = A_2 = A_4 = A_5$ (dotted line). On the bottom: Transverse squared Meissner masses; from top to bottom the lines refer to the gluons $A_6 = A_7$, $A_1 = A_2 = A_4 = A_5$, A_3, and A_8.

where $\vec{q}_I^{\ m}$ ($m = 1, \ldots, N$) are the momenta which define the LOFF crystal relative to the condensate Δ_I; the geometry of the structure and the number N of plane waves should be determined by minimization of the free energy. Once the optimal structure is found, one should compute the Meissner masses. If this structure contains at least three linearly independent momenta, the Meissner tensor should be positive definite for small values of Δ, since it is additive with respect to different terms of (2) to order Δ^2 [17, 18]. These considerations suggest that a LOFF crystal can remove the chromo-magnetic instability of the homogeneous superconductive phases of QCD, resulting as the true vacuum of the theory.

B. Cooling and neutrino emission of a compact star with LOFF matter core

In [19] the specific heat and neutrino emissivity due to direct URCA processes for quark matter in the color superconductive LOFF state with single plane wave pairing are evaluated. The resulting surface cooling curves are shown in Fig. 2. The three lines included in this

figure correspond to three different toy stellar models studied in [19]. The solid line (black online) describes the cooling of a compact star made of electrically neutral nuclear matter of non interacting neutrons, protons and electrons in beta equilibrium; the dashed curve (red online) refers to a toy star with nuclear matter mantle and a core of unpaired quark matter, interacting *via* gluon exchange; the dotted line (blue online) is for a nuclear matter mantle and a core of quark matter in the LOFF state. This last model is computed for $\mu = 500$ MeV and $m_s^2/\mu = 140$ MeV. This results, that should be considered preliminary, apparently show that a toy star with a LOFF matter core seems to cool down faster than an ordinary neutron star. This might have interesting phenomenological consequences.

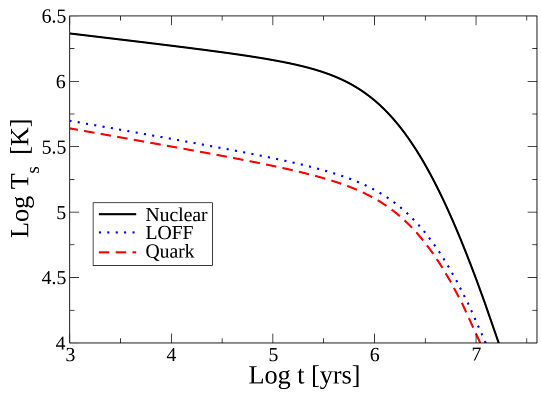

FIG. 2: Surface temperature T_s, in Kelvin, as a function of time, in years, for the three toy models of compact stars described in the text.

III. THREE FLAVOR CRYSTALLINE PHASES

The choice of keeping the wave vectors on the same direction is the simplest, but not necessarily the favored one. This is because the minimization of the thermodynamic potential with respect to the norm of the wave vectors does not give any information about the orientation and the eventual crystalline structure chosen by the system. The only way to determine it is to compare different possible structures and choose the one with lower free energy.

The crystallographic study performed in [16] with a Ginzburg-Landau expansion up to the sixth order in the order parameter Δ reveals two very robust structures, the CubeX and 2Cube45z. Their composition is explained in [16]. Figure 3 shows that they have gaps and free energies as large as one half of corresponding CFL parameters. The figure on the top reveals a first order transition to the normal phase for both the favored structures, within the Ginzburg-Landau analysis, while the figure on the bottom shows that the window in M_s^2/μ, where the free energy of at least one of these structures is lower than the normal and the other color superconducting phases,

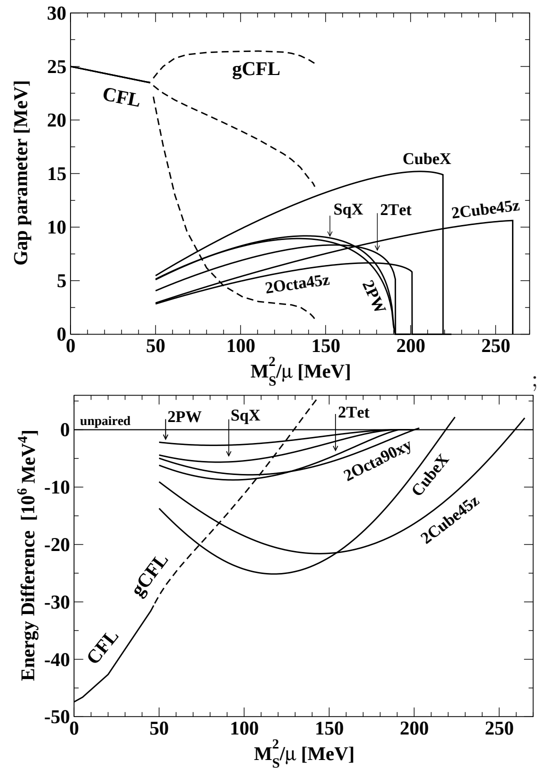

FIG. 3: Gap parameter and Free energy versus M_s^2/μ for three-flavor crystalline color superconducting phases with various crystal structures. The crystal structures are described in [16]. For comparison, the CFL and gCFL corresponding parameters are shown, as found in [4]. The plots are taken from [16].

is broad, e.g.

$$72 \text{ MeV} < m_s^2/\mu < 259 \text{ MeV}\,. \qquad (5)$$

The presence of a first order phase transition, starting from a GL expansion in the order parameter, and the large values for this last one yield to believe that the quantitative results cannot be considered fully reliable. Nevertheless the qualitative picture should be correct, indicating the cubic structures are the preferred ones. Furthermore in the first work of [13] it is shown that the GL analysis tends to underestimate the magnitude of the gap, at least for the one plane wave state.

A. Self-consistent computation of quark masses

The study performed in some works like [20] for normal and homogenous superconducting phases has been extended for 3-flavor LOFF phases in [21], even if a location for these phases into the phase diagram of QCD is not yet possible, since the calculations for these phases are only performed at $T = 0$. The starting point in [21] is a NJL Lagrangean with 4-fermion and 6-fermion quark-antiquark couplings, plus the diquark terms for CubeX

and 2Cube45z crystalline solutions. Treating both the quark-quark and the quark-antiquark interaction in a mean-field approximation, and minimizing the free energy with respect to the chiral ($<\bar{\psi}\psi>$) and the diquark ($<\psi\psi>$) condensates, it is possible to obtain the μ-dependence of the constituent masses of the quarks, where μ is the average chemical potential of the three flavors. The corresponding plot is presented in Fig. 4. Taking in particular the values for the strange quark constituent mass (the constituent masses of *up* and *down* quarks are basically the same as their current masses in the region of interest, since the corresponding chiral symmetry has already been restored), and putting them in the window (5), it is possible to directly obtain the range of μ where the crystalline LOFF phases are favored. This window amounts to be quite large, namely $442\ \mathrm{MeV} < \mu < 515\ \mathrm{MeV}$.

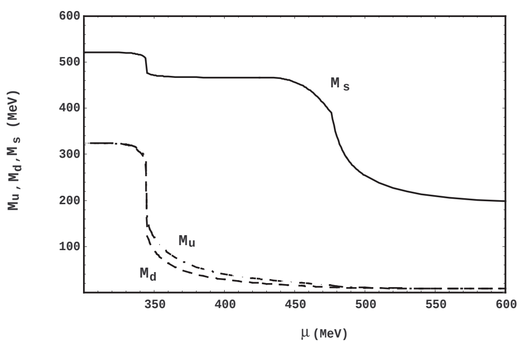

FIG. 4: Constituent quark masses M_s, M_d and M_u as functions of the baryon chemical potential μ.

B. Low energy effective action and masses of pseudo-Goldstones

The breaking of the symmetry group of QCD

$$\mathrm{SU}(3)_c \otimes \mathrm{SU}(3)_V \otimes \mathrm{SU}(3)_A \otimes \mathrm{U}(1)_V \otimes \mathrm{U}(1)_A \qquad (6)$$

in presence of color superconductivity produces a certain number of Goldstone bosons, depending on which symmetries are broken by the particular diquark condensate. In the CFL phase the group is spontaneously broken to the diagonal $\mathrm{SU}(3)_{c+L+R}$ that locks together the color and flavor symmetries, so giving the name to the phase. In the case of three flavor crystalline LOFF phases [22] the color gauge group is again spontaneously broken, so the eight gluons acquire mass by the Anderson-Higgs mechanism, but the presence of nonzero quark masses in the Lagrangian and the difference in chemical potential between the flavors give rise to eight Goldstone modes corresponding to the $\mathrm{SU}(3)_A$ breaking. The breaking of $\mathrm{U}(1)_V$ generates another Goldstone boson, the so-called *superfluid* mode, while it is assumed that the $\mathrm{U}(1)_A$ is restored for the high values of baryon chemical potential considered therein. In [22] the masses and the decay constants for these nine mesons are computed, using a Ginzburg-Landau expansion of the quark propagator up to the second power of $\Delta/\delta\mu$. The main goal of this paper is to study the possibility of meson condensation in LOFF phase, as it occurs in CFL phase. The results at the order $\Delta^2/\delta\mu^2$ show instead that the squared mass matrix for the octet sector is always definite positive, while the superfluid mode is massless. This results exclude the possibility of meson condensation in the LOFF phases with three flavor, but also indicate that the superfluid mode could be important for the low energy spectrum of these phases.

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